

# Element Amplitude Errors

- RMS sidelobes due to amplitude errors
  - Look at power in any direction due to amplitude error

$$P_{AE} = \sum_i (\Delta e_i)^2 = N \cdot (\Delta e)^2 \approx N \cdot \langle (\Delta e)^2 \rangle$$

- Compare to power on main beam

$$P_{Boresight} = \sum_i (e_i)^2 = \sum_i (w_i \cdot e_i)^2 \approx N \cdot \left( \sum_i w_i^2 \right) \cdot \langle e^2 \rangle$$

- Ratio of main beam to RMS sidelobe levels

$$\frac{P_{Boresight}}{P_{AE}} = \left( \sum_i w_i^2 \right) \cdot \frac{\langle e^2 \rangle}{\langle (\Delta e)^2 \rangle}$$

# Element Phase Errors

- Element Phase Errors
  - Due to propagation and element variations as well as phase shifter errors
  - Vary with frequency over the operating band
  - Not necessarily small
  - Vary with phase shift, temperature, time
- Mean amplitude of element with phase error

$$\langle \cos(\theta_e) \rangle = \exp\left(-\frac{\sigma_\theta^2}{2}\right)$$

From a Fourier Transform pair!

# Phase Errors Effects

- Mean Squared Amplitude of Weighted Element

$$\left\langle \left( w_i \cdot \cos(\Delta\theta_i) \right)^2 \right\rangle = w_i^2 \cdot \left( \frac{1 + \exp(-2 \cdot \sigma_\theta^2)}{2} \right)$$

- This Shows a Loss in Antenna Gain
- Total Array Mean Squared Amplitude

$$\sum_i \left\langle \left( w_i \cdot \cos(\Delta\theta_i) \right)^2 \right\rangle = \left( \sum_i w_i^2 \right) \cdot \left( \frac{1 + \exp(-2 \cdot \sigma_\theta^2)}{2} - \exp(-\sigma_\theta^2) \right) \\ + \left( \sum_i w_i \right)^2 \cdot \exp(-\sigma_\theta^2)$$

- Weighting and phase error losses are not totally separable

# Phase Shifter Errors and RMS Sidelobes

- Error in element amplitude

$$\Delta(w_i \cdot e(\Delta\theta_i)) = w_i \cdot 2 \cdot \sin\left(\frac{\Delta\theta_i}{2}\right)$$

- Mean Square Error Signal

$$\left\langle \left( \Delta(w_i \cdot e(\Delta\theta_i)) \right)^2 \right\rangle = w_i^2 \cdot 2 \cdot \left( 1 - \exp\left(-\frac{\sigma_\theta^2}{2}\right) \right)$$

- Not orthogonal in phase to amplitude vector for large errors