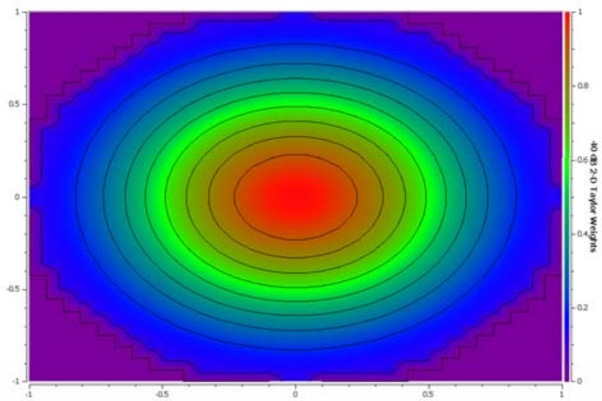

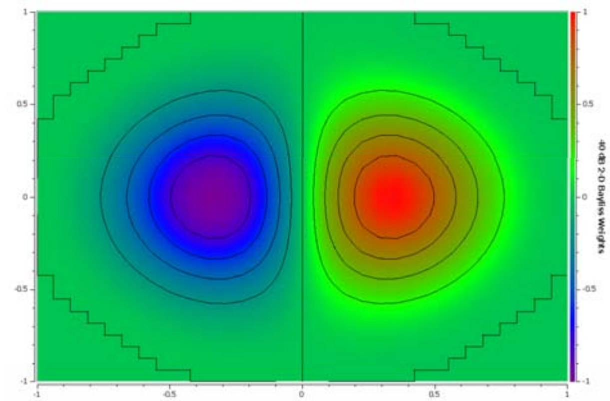

# Planar Array Design and Performance


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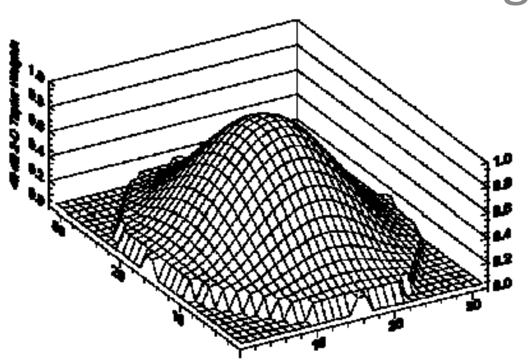

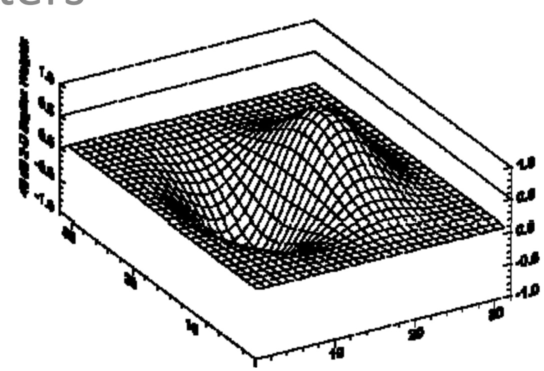

# Today's Topics

- Planar Phased Arrays: Types and Applications
- The Planar Array is a Component of a Sensor System
- The Weightings
  - Dolph-Chebychev – Sidelobe control for square apertures
  - Taylor weighting – Sidelobe control for circular apertures
  - Bayliss weighting – Sidelobe control for difference channels using circular apertures
- The Trade Space and Design Challenges
- Conclusion

# Planar Phased Array Applications

- Communications
  - Satellite based
    - Electrically steered to areas on Earth's surface
    - Sidelobe levels resist competing signals
  - Ground based
    - Electrically steered to LEO and MEO satellites
    - Electrically steered to or from mobile ground stations
- Radar
  - Ground, air and space based
  - Mounted or mechanically steered to field of view
  - Electrically steered to scan and track within FOV

# As a System Component

- Beard’s Theorem
- The top level planar array trade space includes
  - Gain and beamwidth
  - Sidelobe suppression level
  - Element amplitude and phase shifter accuracy and RMS sidelobes
  - Number and spacing of elements
  - Power of transmitter elements
  - Noise figure of receiver elements
  - Antenna or range-Doppler map

# Array Weighting

- Array weighting and the trade space
  - Used on receiver antennas
  - Weighting controls the beam broadening and gain of a phased array antenna
  - Weighting trades out-of-beam directional suppression of unwanted signals for beam broadening and gain reduction
- Limitations on sidelobe suppression
  - Element gain accuracy and phase shifter accuracy limit achievable sidelobe reduction
  - The “noise floor” from these errors is the RMS sidelobe level

[Mathematics of Phase and Amplitude Errors]

# Types of Weighting

- Analytical
  - Chebychev
    - Provides equal sidelobes everywhere in direction cosine space – but element pattern rolls off sidelobes
    - Provides minimum lobe width and highest gain for specified sidelobe level
    - Requires square or rectangular grid
    - High performance Chebychev weighting is nearly zero outside a circle
    - Exhibit "Baklanov ring" sidelobes (LINK)
  - Other
    - Can provide split main beam for angle finding
    - Disadvantages similar to those of Chebychev weighting
- Optimization
  - Linear Programming
  - Quadratic Cost Functions
- Heuristic
  - Taylor – essentially optimal circular sampled continuous weight
  - Bayliss – essentially optimal circular sampled continuous weight that provides split main lobe for angle finding

# Taylor Window

- Circular Apertures Sampled by Discrete Elements
- Synthesized pattern in direction cosine "u-v" space by using
  - Near the main beam, a scaled objective function with ideal pattern
  - Away from the main beam, an asymptotic function that
    - Contains weighting within a circular aperture
    - Provides mathematically feasible spatial domain weighting
  - Spatial domain weighting found by Fourier-Bessel transform of this *ad hoc* pattern
- Mathematics is dominated by Fourier-Bessel Transform

Mathematics of Taylor Window

# Bayliss Weighting

- Objective Function
  - Found by taking derivative of limiting form of Chebychev frequency response with respect to one direction cosine
  - Or by using Chebychev polynomial of second kind in formulation
- Asymptotic Function
  - Array weight is simple tilted plane
  - Bessel transform is first derivative of $J_1(x)$
    - Simple
    - Asymmetrical
    - Realizable

Mathematics of Bayliss Window

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# Examples of Aperture Weightings

| | | | | | | |
|---|---|---|---|---|---|---|
| **Chebychev** | Weights Fishnet | Weights Contour | Beam Pattern Fishnet | Beam Pattern Contour | Main Lobe | Noise Performance Curves |
| **Taylor** | Weights Fishnet | Weights Contour | Beam Pattern Fishnet | Beam Pattern Contour | Main Lobe | Noise Performance Curves |
| **Bayliss** | Weights Fishnet | Weights Contour | Beam Pattern Fishnet | Beam Pattern Contour | Main Lobe | Noise Performance Curves |

Analysis Program

Development Environment

# Sampling Errors

- Sampling the Taylor or Bayliss Apertures
  - Sampling theory
    - RMS sidelobe level is driven by edges in Taylor weightings
    - Errors increase with faster variation in weighting functions; these are analogous to increased bandwidth of sampled signals
    - Errors vary inversely with number of samples across the aperture
  - Errors vary with sampling scheme
    - Classical square spacing is simplest to analyze
    - Commonly used equilateral triangular spacing appears to be optimal for grating lobes

Examples of Sampling Errors

# Element Placement

- Types of element placement
  - Square – equal vertical and horizontal spacing
  - Rectangular – still on a 90 degree grid but unequal spacing vertically and horizontally
  - Triangular – equilateral or isosceles triangular pattern
- Trade space
  - Density
    - Number of elements
    - Grating lobes
  - Cost increases with number of elements

# Grating Lobes

- Grating Lobes
  - Named after analogous optical phenomena with diffraction gratings, Fresnel lenses, etc.
  - Steer with the main lobe, configuration with main lobe remains constant in sine space
  - Occur at any angle characterized as normal to parallel lines of elements through the array
  - Distance from main lobe is inversely proportional to distance between the parallel lines
  - Common at multiples of 45 degrees in square spacing
  - Common at multiples of 30 degrees in triangular spacing
  - Must occur in "real" sine space, $|u + j \cdot v| \leq 1$ for some beam steering to manifest in real world
- Increasing Element Spacing to put Real Grating Lobes near Element Pattern Nulls
  - Transmitted power is not seen at or near element nulls in the far field pattern
  - Installed element nulls often are not in the same place as isolated element nulls
  - Power that the array pattern transmits near element nulls will increase observed VSWR
  - Power not transmitted is absorbed by dummy loads on the circulators – hence Eli Brookner's Law: Radars tend to catch fire.
- Summary
  - Grating lobes are always there
  - Contrive a 2-D FFT with your element placement and see where they ALL are
  - Remediation: Keep them out of the unit circle for planned steering, or place them in element nulls

# Time Delay Beamforming

- Phase Shift Beamforming
  - Filters the signal in steered beams with a weighted averager
  - Total duration is $(D/c) \cdot \sin(\psi)$
  - Phase shift beamforming MAY be OK if $BW \cdot D/c \ll 1$
- Common compromise
  - Phase shift beaming over subarrays
  - Time delay beamforming among subarrays
- Antenna frequency response versus angle is a system consideration for wideband signals

# The Element Trade Space

- Elements
  - Size of array in wavelengths
  - Number of elements
  - Amplitude accuracy
  - Phase shifter accuracy: the EITLR

$$\sigma_\theta \approx \frac{0.6 \cdot \sqrt{N}}{10^{(\text{dB Down})/20}} (\text{radians})$$

- As Aperture Weighting Sidelobes Decrease
  - Noise bandwidth increases
  - Array efficiency decreases

Element Phase Error Comparison

# Array Noise Bandwidth?

- Term in analogous to FIR filter noise bandwidth
  - Equal to normalized sum of squares of weights
  - Represents the response of the array to isotropic white noise; related to decrease in directional gain
  - Band limiting in the receiver makes actual numbers application-dependent
- This is a number useful in evaluating power into the beamforming sum in isotropic noise

Noise Bandwidth Comparison

# Array Efficiency

- Equal to RMS element amplitude weighting
  - Equal to 1.00 for unweighted array
  - Represents loss of ERP on boresight due to amplitude reduction of some of the elements
- Array Efficiency is the Maximum Peak ERP as a Fraction of Maximum Possible Total Power
- Related to Antenna Gain

Array Efficiency Comparison

# Matching Up the Requirements

- The Design Parameters
  - Select Taylor and Bayliss Aperture Weightings
    - Sidelobe suppression level
    - Antenna gain and ERP at boresight
    - Monopulse accuracy and angle extent in sine space
  - Select Elements and Determine Free and Installed Element Patterns
    - Element pattern and field of view for ESA
  - Select Element Placing and Number of Elements
  - Determine Element Control Requirements
    - Amplitude RMS accuracy requirement
    - Phase RMS accuracy requirement
- Antenna frequency response versus angle off axis
  - Verify sidelobe performance
  - Verify any effect of phase shift beamforming on processing losses
  - Determine time delay beamforming requirements

# Achieving the Required RMS Element Phase Errors

- A Five-Bit Phase Shifter
  - Provides a phase quantization of 11.25 Degrees
  - RMS Quantization Error over Desired Phase is Quantization Interval Divided by SQRT(12) or 3.25 Degrees
  - This is 0.057 Radians, Which Supports 60 dB Sidelobe Suppression (see RMS Element Phase Error Comparison)
- Design Challenge
  - RMS Phase Accuracy of 3 Degrees or 0.052 Radians
    - Across operating band for each mode
    - Across field of view
    - For all phase shifter settings
  - Amplitude accuracy to match
  - Maintain calibration over time and temperature

# Element Design Challenges

- System Noise Budget
  - Power Supply Noise and Ripple vs. Voltage Pushing Factors, EMC coupling, etc. on Amplitude and Phase
    - Crystal or fundamental frequency source
    - Exciter synthesizer chains
    - Transmitter amplification chain
    - Transmitter DAC
    - Receiver LNA
    - Receiver Amplification, BFO phase noise, Mixer, etc.
    - ADC amplitude scaling
    - ADC Track-and-Hold Clock and its clock exciter chain
- Match Element RMS Phase and Amplitude Accuracy

# Antenna Range Issues

- Indoor "Quiet Room"
  - Excellent for developing and calibrating elements
  - Excellent for near-field calibration of arrays
  - Excellent for steered beams, far sidelobes
  - Reflected RF Must Support Sidelobe Levels
- Boresight Towers
  - Excellent for maintaining calibration and performance for ground radars
  - Most practical for ground-based platforms
  - Measure only one true az/el unless something moves

# High Performance Raises Element Calibration Issues

- Active Array Element VSWR
  - Can vary with
    - Angle of incidence
    - Temperature
    - Time
  - Will impact insertion phase and gain
- Temperature Changes in Usage
  - Varies with transmit power
  - Varies over the array
  - Thermal control cannot be completely uniform
  - Temperature will vary insertion phase and gain

# Digital-Intensive Signal Processing Architectures

- Rank 2 data stream in signal processing
  - Range-Doppler map
  - Video imagery
  - High resolution CAT scans
- Aperture weighting including time dimension
  - Weight of each data sample is a function of each of three coordinates
  - Dependence on time variable means that aperture weighting is time dependent
- Scalloping loss, weighting overlap, processing losses trade space depend on weighting parameters

# Range-Doppler-Time?

- Three Dimensions
  - Range, Doppler, Time
  - Bayliss splits azimuth, elevation, and time
  - Replaces spectral weighting on FFT
  - Requires spheroidal wave functions
- Spheroidal wave functions
  - Classical theory exists for oblate (disk-like) and prolate (cigar-like) spheroids of rotation
  - Applicable for adding time dimension
  - Cannot address elliptical apertures AND time dimension

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# A Three-D FFT?

- Advantages
  - Uses range-Doppler-time weighting as an FFT weighting
  - Can use two-dimensional aperture weighting and separate FFT after beamforming
  - Can use spheroidal weighting
  - Can implement true-time-delay beamforming as linear phase shift versus position
  - Can use three-dimensional autocorrelation to track target movement during averaging time without additional processing losses
  - Opportunity for convolution with signal
- Disadvantages
  - Requires separate LNA, ADC for each element or subarray
  - True-time-delay beamforming requires a separate 3-D FFT for each beam

# What About Elliptical Apertures?

- The Taylor and Bayliss Weights
  - Classical Taylor and Bayliss weights empirically do not work for elliptical apertures
  - Extension of theory is straightforward but requires use of elliptical wave (Mathieu) functions
  - Mathieu functions are far more complex to deal with
- Wave Theory in Elliptical Coordinates
  - Based on elliptical wave functions == Mathieu functions
  - Evaluated in elliptical-hyperbolic coordinates
  - As eccentricity decreases to zero, become Bessel functions and circular functions

# Four-Lobe "Difference" Channel?

- Explored by Chesley (IEEE Trans. Ant. & Prop., Oct 1992 pp 1187-1191)
- Treatment varied Bayliss' by doubling angle dependency and using $J_2(r)$ instead of $J_1(r)$
- N Bar problem of Bayliss not mentioned
- Application was ECCM for jammers in the main beam
  - Target angle finding not addressed
  - Monopulse would require two – or three – channels
  - Offers potential of nulling jammers in the main beam

# Conclusions

- Hardware State of the Art May Support 60+ dB out-of-beam Rejection
  - Detection Beam
  - Difference Beams
- Rank 2 Computed Data Streams are Limited Only by the System Error Budget
  - Range-Doppler maps
- The System Error Budget Must Evolve to Support Higher Out-of-Beam Rejection Performance
- Extensions Required for High Performance Taylor and Bayliss Aperture Weighting First Reported Here
- Moore's Law Puts More Sophisticated Architectures On the Table

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